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Investigation of effects of wave motion on the performance of a falling film absorber

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Abstract—The effect of wave motion on the performance of the falling film absorbers was analytically investigated by solving the combined energy and diffusion equations at a low Reynold's number (*Re* = 100) using an Alternating Direction Implicit (ADI) finite difference method. By comparing the results of smooth flow and wavy flow it was found that wave motion improved the heat and mass transfer rates despite using a film velocity profile that did not permit mixing. The results supported the findings of other researchers who explained the enhancement of mass transfer at low *Re* by the convection associated with the vertical component of the velocity. The effects of heat of absorption (*Ha*) and Lewis number (*Le*) on heat and mass transfer were also studied and it was found that a low value of *Ha* together with a high *Le* would result in a better concentration variation. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

In absorption refrigeration systems the absorber has long been recognised as the most important and critical component. Absorbers, in which a thin film of absorbent is falling freely over a bundle of horizontal or vertical tubes or over cooled plates are widely used. They achieve high effectiveness by virtue of the thinness of the absorbent's flow [1] which maximizes the ratio of the surface area to mass flow rate and minimizes the penetration length, hence aids the process of both heat transfer to the coolant and mass transfer of the vapour refrigerant.

A good body of literature is available on the nature of waves in film flow regimes. Having studied the wave motion photographically Stainthorp and Allen [2] concluded that wave characteristics depend on the flow rate, the physical properties of the liquid as well as the distance from the leading edge. Using laser technique Salazar and Marshall [3] observed that waves were primarily of two-dimensional nature up to Reynold's number, *Re* = 375. The survey of Rotem and Neilson [4] suggested that there were three distinct regimes of film flow depending on Reynold's number:

laminar flow without rippling of the free surface at *Re* < 7; laminar flow with rippling (wavy) at 7 < *Re* < 250 and turbulent flow at *Re* > 250.

In the applications of absorption refrigeration it is unlikely to have the values of *Re* exceeding 300 [5] and the flows are therefore likely to be laminar wavy [4, 6, 7]. Previous researchers [8–10] noticed the significant improvement to heat and mass transfer due to the presence of waves in the thin film flow regimes. With sufficiently high Reynold's number, the improvements of heat and mass transfer were the results of the mixing effect of the waves [11–13]. At low *Re*, however, where the mixing effects are less pronounced or do not exist, further explanation for the improvements is needed. By assuming that at low *Re* waves would be regular and two-dimensional, Penev *et al.* [14] solves the Navier–Stokes equation for a falling liquid film. They found that under such conditions the liquid elements travelled along unclosed trajectories and there could be no surface renewal (mixing), and the improvement in mass transfer was primarily due to the vertical component of the film velocity. This component, which did not exist in the smooth flow regimes, caused the vapour to be convected across

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NOMENCLATURE

A	dimensionless wave amplitude, $A = (h_{\max} - h_0)/h_0$
$a(\varepsilon)$	coefficient used in equation (53)
a, $\tilde{a}$	coefficients used in equation (54)
$\acute{a}(\varepsilon)$	differential of $a(\varepsilon)$ with respect to ε
b, d	coefficients used in equation (59)
c	mass concentration of the liquid [kg kg^{-1}]
C_p	heat capacity [kJ kg^{-1} °C^{-1}]
D	diffusion coefficient [m^2 s^{-1}]
Δh	specific enthalpy of evaporation [kJ kg^{-1}]
Ha	dimensionless heat of absorption
h_0	average film thickness [m]
$h(\varepsilon)$	local film thickness [m]
$H(\varepsilon)$	dimensionless film thickness, $H(\varepsilon) = h(\varepsilon)/h_0$
K	thermal conductivity [W m^{-1} °C^{-1}]
Le	Lewis number, $Le = D/\alpha$
M	number of elements in the X direction
N	number of elements in the Y direction
n	dimensionless wave number; $n = 2\pi h_0/\lambda$
Pe	Peclet number, $Pe = v_0 h_0/D$ or $Pe = (Re)(Sc)$
Re	Reynolds number, $Re = \Gamma/\rho v$
Sc	Schmidt number, $Sc = v/D$
t	time [s]
t_w	periodical time [s]
T	local temperature [°C]
v_0	average velocity of the liquid [m s^{-1}]
v_x	local velocity of the liquid along the x axis [m s^{-1}]
V_x	dimensionless velocity of the liquid in the direction of x
v_y	local velocity of the liquid along the y axis [m s^{-1}]
V_y	dimensionless velocity of the liquid in the direction of y
x, y	coordinates in x and y direction, respectively
X, Y	dimensionless x and y coordinates, respectively
Z	dimensionless vertical coordinate in the wavy field.

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
γ, θ	dimensionless concentration and temperature, respectively
Γ	mass flow rate per unit width [kg m^{-1} s^{-1}]
$\Delta\tau$	dimensionless time step
ΔX, ΔY, ΔZ	dimensionless step in X, Y and Z, respectively
ε	parameters, $\varepsilon = (x - \omega v_0 t)/h_0$
λ	wave length [m]
v	kinematic viscosity of the liquid [m^2 s^{-1}]
ρ	density [kg m^{-3}]
τ	dimensionless time, $\tau = Dt/h_0^2$
τ_w	dimensionless periodical time
ω	dimensionless phase velocity.

Subscripts

0	initial time condition
0, 1, 2, . . .	index number of elements
eq	equilibrium condition
i	initial space (inlet) condition
i, j	indices for the ith and jth elements, respectively
w	condition of the wave or at the wall
X, Y, Z	respective components in the X, Y and Z directions or index of partial differentials with respect to X, Y and Z, respectively.

Superscripts

1, 2, . . . k	index for time elements
*	index for half time step.

the vapour–liquid interface thus improved its transfer rate. In investigating a mass transfer problem Beschkov and Boyadjiev [15] employed a finite difference method to solve the equation of vapour diffusion into a thin liquid film flowing in a laminar wavy manner. They used the values computed by Penev *et al.* [14] for the velocity profile (which denied any mixing possibility) and wave characteristics, and obtained data on mass transfer which agreed well with their own experimental work. Ibrahim [16] suggested that for flows with $Re < 100$ a model with regular wave characteristics and no mixing should be applied to estimate the improvement in heat and mass transfer.

It was felt that the attribution of improvement of heat and mass transfer processes solely to the existence of the vertical component of the velocity is not well understood and there seemed to be little work done on problems of combined heat and mass transfer in thin film wavy flow at low Re. The data of Penev *et al.* [14], who devised a velocity profile that denies the possibility of mixing [15], was used in this investigation to evaluate the heat and mass transfer pro-

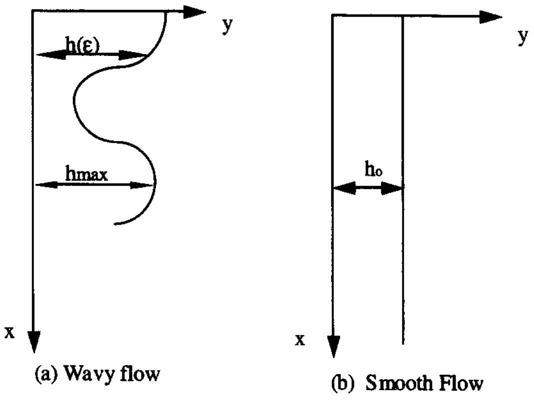

Fig. 1. The wavy and smooth flow domain.

cesses associated with the absorption of a water vapour by a thin wavy laminar film of LiBr solution flowing downwards under the action of gravity.

ANALYSIS

For a thin wavy liquid film flowing down a vertical (or inclined) surface under the system of coordinates shown in Fig. 1(a), the diffusion and energy equations [17] can be written respectively as

$$\frac{\partial c}{\partial t}+v_x\frac{\partial c}{\partial x}+v_y\frac{\partial c}{\partial y}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right) \quad (1)$$

$$\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \quad (2)$$

Initial and boundary conditions

To solve equations (1) and (2), the initial and boundary conditions must first be obtained. The initial distributions of concentration and temperature are assumed to be of the smooth flow, i.e.

$$\text{at time } t=0 \quad c=c_0(x,y) \quad \text{and} \quad T=T_0(x,y).$$

The boundary conditions at inlet are assumed to be well mixed and uniform, i.e.

$$\text{at } x=0 \quad c=c_i \quad \text{and} \quad T=T_i.$$

For the y coordinate the concentration boundary condition at the wall reflects the impermeability of the solid wall, i.e.

$$\text{at } y=0 \quad \frac{\partial c}{\partial y}=0$$

and the temperature is assumed to be equal to the wall temperature, i.e.

$$T=T_w.$$

At the liquid–vapour interface the boundary conditions assume that the absorbent is saturated and can be approximated by the linear dependency of concentration and temperature [16, 18] as shown below:

$$\text{at } y=h(\varepsilon) \quad T=ac+b \quad (3)$$

where a and b are constants.

At the interface, if the sensible heat transfer is neglected, the heat flux is proportional to the rate of vapour absorption [16], i.e.

$$\text{at } y=h(\varepsilon) \quad \frac{\partial \theta}{\partial y}=Ha\frac{\partial \gamma}{\partial y}$$

where θ, γ and Ha are the dimensionless temperature, concentration and heat of absorption defined respectively as:

$$\gamma=\frac{c-c_i}{c_{eq}-c_i} \quad \theta=\frac{T-T_i}{T_{eq}-T_i}$$

$$Ha=\left(\frac{\rho D\,\Delta h}{K}\right)\left(\frac{c_{eq}-c_i}{T_{eq}-T_i}\right).$$

Smooth flow (initial condition)

It is necessary to find the initial distributions of concentration $c_0(x,y)$ and temperature $T_0(x,y)$ defined previously not only as initial conditions of the time dependant wavy distributions, but also because both smooth and wavy distributions will be compared to each other to assess the relative improvements in heat and mass transfer brought about by the actions of waves.

The diffusion and energy equations for a smooth flow under the same conditions and system of coordinates, shown in Fig. 1(b), as those of the wavy flow described earlier, can be written as:

$$v_x\frac{\partial c}{\partial x}=D\frac{\partial^2 c}{\partial y^2} \quad (4)$$

$$v_x\frac{\partial T}{\partial x}=\alpha\frac{\partial^2 T}{\partial y^2}. \quad (5)$$

By defining and using the following dimensionless properties

$$X=\frac{x}{Peh_0},\quad Y=\frac{y}{h_0},\quad V_x=\frac{v_x}{v_0}$$

equations (4) and (5) can be transformed into non-dimensional form as:

$$V_x\frac{\partial \gamma}{\partial X}=\frac{\partial^2 \gamma}{\partial Y^2} \quad (6)$$

$$V_x\frac{\partial \theta}{\partial X}=\frac{1}{Le}\frac{\partial^2 \theta}{\partial Y^2}. \quad (7)$$

The liquid domain is divided into a mesh of small elements with the grid points designated by i and j along the x and y axes, respectively, where $i=1,2,3,\ldots M$ and $j=1,2,3,\ldots N$. An explicit finite difference scheme was used to approximate the differential equations (6) and (7), which accordingly becomes

$$\gamma_{i+1,j} = \mathrm{Rc}\left(\gamma_{i,j+1}+\gamma_{i,j-1}-\left(2-\frac{1}{\mathrm{Rc}}\right)\gamma_{i,j}\right) \quad (8)$$

$$\theta_{i+1,j} = \mathrm{RT}\left(\theta_{i,j+1}+\theta_{i,j-1}-\left(2-\frac{1}{\mathrm{RT}}\right)\theta_{i,j}\right) \quad (9)$$

where

$$\mathrm{Rc} = \frac{\Delta X}{V_x(\Delta Y)^2} \quad \text{and} \quad \mathrm{RT} = \frac{\Delta X}{LeV_x(\Delta Y)^2}.$$

Explicit finite difference schemes are simple to construct and solve, but have the drawback that ΔX and ΔY have to be chosen to satisfy certain stability criterion; Rc and RT must everywhere be no greater than 0.5 which implies a smaller mesh size than in the case of implicit methods, hence a longer calculation time. This is not a disadvantage in this particular problem because, as mentioned earlier, the smooth distributions of concentration and temperature will be used as the initial condition for the wavy distribution and will be compared to it to obtain the relative improvement, thus the mesh size has to be the same in both cases, and with the complicated stability requirements of the wavy problem, its mesh size is very small anyway. Therefore using the implicit method to increase the mesh size of the smooth flow domain would not be of any benefit.

Since $\mathrm{RT} = \mathrm{Rc}/Le$ and Le is less than unit, therefore $\mathrm{RT} > \mathrm{Rc}$, and consequently if RT is no greater than 0.5 neither will be Rc. To calculate the mesh size it is noted that since $Y = y/h_0$ and in the non-wavy flow: $y_{\max} = h_0$

$$\text{then} \quad Y_{\max} = \frac{y_{\max}}{h_0} = 1$$

and so

$$\Delta Y = \frac{1}{N}.$$

Following Penev *et al.* [14] a half-parabolic velocity profile is assumed

$$V_x = \frac{v_x}{v_0} = \tfrac{3}{2}(2Y-Y^2).$$

The maximum possible value of RT is given by

$$\mathrm{RT}_{\max} = \frac{\Delta X}{(LeV_{x(\min)}(\Delta Y)^2)}$$

where $V_{x(\min)} = \frac{3}{2}(2\Delta \mathrm{Y}-(\Delta \mathrm{Y})^2)$, occurring at the element adjacent to the solid wall. Equating $\mathrm{RT}_{\max}$ to 0.5 enables the calculation of ΔX which produces a stable solution, i.e.

$$\Delta X = 0.75Le(2(\Delta Y)^3-(\Delta Y)^4).$$

Smooth flow (*boundary conditions*)

The boundary conditions for the non-wavy flow are the same as those of the wavy flow. They are written for the dimensionless domain as follows:

$$\text{at } x = 0, \quad X = 0, \quad c = c_i, \quad \gamma_{0,j} = 0$$

$$\text{and} \quad T = T_i, \quad \theta_{0,j} = 0$$

$$\text{at } y = 0, \quad Y = 0, \quad \frac{\partial c}{\partial y} = \frac{(c_{eq}-c_i)}{h_0}\frac{\partial \gamma}{\partial Y} = 0$$

i.e. $\partial\gamma/\partial Y = 0$, or in finite difference form

$$\frac{(\gamma_{i,1}-\gamma_{i,0})}{\Delta Y} = 0.$$

It was mentioned earlier that the temperature of the liquid element at $y = 0$ is taken to be equal to the wall temperature (T_w). For simplicity T_w was assumed to be constant and equal to T_i, i.e. isothermal cooling. The cooling mode makes no difference to the relative result as it is to be applied to both smooth and wavy flow. Therefore

$$\text{at } Y = 0, \theta_{i,0} = \theta_{0,j} = \theta_w = 0$$

and at $y = h_0$, $Y = 1$ equation (3) states that

$$T = ac+b.$$

This equation can be transformed into the dimensionless form [16] as:

$$\gamma+\theta = 1$$

and is written for the interface as:

$$\gamma_{i,N}+\theta_{i,N} = 1.$$

Also at the interface

$$\frac{\partial \theta}{\partial Y} = Ha\frac{\partial \gamma}{\partial Y}$$

which can be written in dimensionless interface notation as:

$$\theta_{i,N}-\theta_{i,N-1} = Ha(\gamma_{i,N}-\gamma_{i,N-1}).$$

Solving the finite difference equations (8) and (9) gives the distribution of concentration and temperature in the smooth film which will be used as the initial distribution ($t = 0$) for the wavy problem. Both solutions of smooth and wavy will later be compared.

Wavy distribution

The main difficulty in solving the problem of the wavy flow is that the film surface is rippled and so the film thickness $h(\varepsilon)$ varies temporarily as well as spatially along the film length. This difficulty was overcome by introducing a new variable Z, where

$$Z = \frac{y}{h(\varepsilon)} \quad 0 \leqslant Z \leqslant 1.$$

Hence the wavy domain (x, y, t) could be transformed into, in effect, the smooth domain (x, Z, t). The following notations for partial derivative are used for the subsequent mathematical manipulation of the transformation.

$$\frac{\partial}{\partial x}f(x,y,t)=\frac{\partial f}{\partial x} \quad \text{and} \quad \frac{\partial}{\partial x}f(x,Z,t)=f'_x, \quad \text{etc.}$$

Therefore

$$\mathrm{d}f=\frac{\partial d}{\partial x}\mathrm{d}x+\frac{\partial f}{\partial y}\mathrm{d}y+\frac{\partial f}{\partial t}\mathrm{d}t \tag{10}$$

and

$$\mathrm{d}f=f'_x\,\mathrm{d}x+f'_Z\,\mathrm{d}Z+f'_t\,\mathrm{d}t \tag{11}$$

where

$$\mathrm{d}Z=\frac{\partial Z}{\partial x}\mathrm{d}x+\frac{\partial Z}{\partial y}\mathrm{d}y+\frac{\partial Z}{\partial t}\mathrm{d}t. \tag{12}$$

Therefore

$$\mathrm{d}f=\left(f'_x+f'_Z\frac{\partial Z}{\partial x}\right)\mathrm{d}x+f'_Z\frac{\partial Z}{\partial y}\mathrm{d}y+\left(f'_t+f'_Z\frac{\partial Z}{\partial t}\right)\mathrm{d}t \tag{13}$$

and equating the coefficients of equations (10) and (13) one obtains

$$\frac{\partial f}{\partial x}=f'_x+f'_Z\frac{\partial Z}{\partial x}$$

$$\frac{\partial f}{\partial y}=f'_Z\frac{\partial Z}{\partial y}$$

$$\frac{\partial f}{\partial t}=f'_t+f'_Z\frac{\partial Z}{\partial t}$$

$$\frac{\partial^2 f}{\partial x^2}=f''_{xx}+f''_{Zx}\frac{\partial Z}{\partial x}+f'_Z\left(\frac{\partial Z}{\partial x}\right)_x + \left(f''_{xZ}+f''_{ZZ}\frac{\partial Z}{\partial x}+f'_Z\left(\frac{\partial Z}{\partial x}\right)_Z\right)\frac{\partial Z}{\partial x}.$$

Since neither f'_Z nor $\partial Z/\partial y$ is function of y and $\partial Z/\partial y$ is only a function of x and t, therefore

$$\left(f'_Z\frac{\partial Z}{\partial y}\right)_y=0 \quad \text{and} \quad \left(\frac{\partial Z}{\partial y}\right)_Z=0$$

and

$$\frac{\partial^2 f}{\partial y^2}=\left(f'_Z\frac{\partial Z}{\partial y}\right)_y+\left(f'_Z\frac{\partial Z}{\partial y}\right)_Z\frac{\partial Z}{\partial y}=f''_{ZZ}\left(\frac{\partial Z}{\partial y}\right)^2.$$

Equations (1) and (2) can both be expressed in a general form:

$$\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y}=\text{Constant}\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\right) \tag{14}$$

where f is the concentration function, c, and the constant represents the D in the diffusion coefficient in the diffusion equation, equation (1), or f is the temperature function, T, and the constant refers to thermal diffusivity, α, in the energy equation, equation (2).

Substituting for the derivatives, the left-hand side of equation (14) becomes

$$\begin{aligned}\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+v_y\frac{\partial f}{\partial y} &= f'_t+f'_Z\frac{\partial Z}{\partial t}+v_x\left(f'_x+f'_Z\frac{\partial Z}{\partial x}\right)+v_yf'_Z\frac{\partial Z}{\partial y}\\ &= f'_t+v_xf'_x+\frac{\mathrm{d}Z}{\mathrm{d}t}f'_Z.\end{aligned} \tag{15}$$

Using the 2nd order derivatives the right-hand side of equation (14) gives

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=f''_{xx}+2f''_{xZ}\frac{\partial Z}{\partial x} +f''_{ZZ}\left(\frac{\partial Z}{\partial x}\right)^2+f''_{ZZ}\left(\frac{\partial Z}{\partial y}\right)^2+f'_Z\left(\frac{\partial Z}{\partial x}\right)_x. \tag{16}$$

Equating the right-hand sides of equations (15) and (16) produced the general diffusion and energy equation in the transformed domain (x, Z, t).

$$\frac{\partial f}{\partial t}+v_x\frac{\partial f}{\partial x}+\frac{\mathrm{d}Z}{\mathrm{d}t}\frac{\partial f}{\partial Z} = \text{Constant}\left(\frac{\partial^2 f}{\partial x^2}+2\frac{\partial^2 f}{\partial x\,\partial Z}\frac{\partial Z}{\partial x} +\frac{\partial^2 f}{\partial Z^2}\left(\frac{\partial Z}{\partial x}\right)^2+\frac{\partial^2 f}{\partial Z^2}\left(\frac{\partial Z}{\partial y}\right)^2+\frac{\partial f}{\partial Z}\frac{\partial^2 Z}{\partial x^2}\right). \tag{17}$$

By defining a dimensionless time τ as $\tau = Dt/h_0^2$, equation (17) can be transformed into non-dimensional form, written as:

$$\frac{\partial f}{\partial \tau}+V_x\frac{\partial f}{\partial X}+\left(\frac{\mathrm{d}Z}{\mathrm{d}\tau}-B\left(\frac{\partial^2 Z/\partial X^2}{Pe^2}\right)\right)\frac{\partial f}{\partial Z} = B\left(\frac{1}{Pe^2}\frac{\partial^2 f}{\partial X^2}+\left(\frac{1}{H(\varepsilon)^2}+\left(\frac{\partial Z/\partial X}{Pe}\right)^2\right)\frac{\partial^2 f}{\partial Z^2} +2\frac{\partial^2 f}{\partial X\,\partial Z}\frac{\partial Z/\partial X}{Pe^2}\right) \tag{18}$$

where Pe is defined as $Pe = h_0v_0/D$, $B = \text{constant}/D$ and $H(\varepsilon)$ is the non-dimensional film thickness.

In the particular problem concerned Peclet number is large ($Pe = 25.0\text{E}8$) and therefore equation (18) can be simplified by neglecting terms containing Pe^2 in the denominator, and it becomes

$$\frac{\partial f}{\partial \tau}+V_x\frac{\partial f}{\partial X}+\left(\frac{\mathrm{d}Z}{\mathrm{d}\tau}\right)\frac{\partial f}{\partial Z}=B\left(\frac{1}{H(\varepsilon)^2}\frac{\partial^2 f}{\partial Z^2}\right).$$

By expressing $B = D/D = 1$ and $B = \alpha/D = 1/Le$ the diffusion and the energy equation, in non-dimensional form, can be written respectively as:

$$\frac{\partial \gamma}{\partial \tau}+V_x\frac{\partial \gamma}{\partial X}+\frac{\mathrm{d}Z}{\mathrm{d}\tau}\frac{\partial \gamma}{\partial Z}=\frac{1}{H(\varepsilon)^2}\frac{\partial^2 \gamma}{\partial Z^2} \tag{19}$$

$$\frac{\partial\theta}{\partial\tau}+V_x\frac{\partial\theta}{\partial X}+\frac{dZ}{d\tau}\frac{\partial\theta}{\partial Z}=\frac{1}{Le}\frac{1}{H(\varepsilon)^2}\frac{\partial^2\theta}{\partial Z^2}. \qquad (20)$$

SOLUTION

To obtain the solution for equations (19) and (20) the quantities V_x, $dZ/d\tau$ and $H(\varepsilon)$ must be first evaluated. The velocity profile V_x across the Y axis is calculated from the velocity profile of Penev *et al.* [14]:

$$V_x = a_1(\varepsilon)Y+a_2(\varepsilon)Y^2 \qquad V_x=\sum_{k=1}^{2}a_k(\varepsilon)Y^k \qquad (21)$$

where,

$$a_1(\varepsilon) = a_{1,0}+a_{1,1}\sin(n\varepsilon)+\tilde{a}_{1,1}\cos(n\varepsilon) +a_{1,2}\sin(2n\varepsilon)+\tilde{a}_{1,2}\cos(2n\varepsilon) \qquad (22)$$

$$a_2(\varepsilon) = a_{2,0}+a_{2,1}\sin(n\varepsilon)+\tilde{a}_{2,1}\cos(n\varepsilon) +a_{2,2}\sin(2n\varepsilon)+\tilde{a}_{2,2}\cos(2n\varepsilon) \qquad (23)$$

where ε is expressed in non-dimensional form as

$$\varepsilon=\frac{\left(XPeh_0-\dfrac{\omega v_0 h_0^2\tau}{D}\right)}{h_0}=(X-\omega\tau)Pe.$$

Dividing equation (12) by dt gives

$$\frac{dZ}{dt}=\frac{\partial Z}{\partial t}+v_x\frac{\partial Z}{\partial x}+v_y\frac{\partial Z}{\partial y}. \qquad (24)$$

By defining V_y, the non-dimensional velocity in y direction, as

$$V_y=\frac{Pe}{v_0}v_y$$

and substituting into equation (24) together with the other defined non-dimensional quantities gives

$$\frac{dZ}{d\tau}\frac{D}{h_0^2}=\frac{\partial Z}{\partial\tau}\frac{D}{h_0^2}+V_x\frac{v_0}{Peh_0}\frac{\partial Z}{\partial X}+V_y\frac{v_0}{Peh_0}\frac{\partial Z}{\partial Y}$$

multiplying by h_0^2/D one obtains

$$\frac{dZ}{d\tau}=\frac{\partial Z}{\partial\tau}+V_x\frac{\partial Z}{\partial X}+V_y\frac{\partial Z}{\partial Y} \qquad (25)$$

and since

$$Z=\frac{y}{h(\varepsilon)} \quad\text{and}\quad Y=\frac{y}{h_0}$$

therefore

$$\frac{\partial Z}{\partial Y}=\frac{1}{H(\varepsilon)}$$

$$\frac{\partial Z}{\partial\tau}=\frac{-Y\dfrac{\partial H(\varepsilon)}{\partial\tau}}{H(\varepsilon)^2}. \qquad (26)$$

The local film thickness is given by Penev *et al.* [14] as:

$$h(\varepsilon)=h_0[1+A\sin(n\varepsilon)+b_2\sin(2n\varepsilon) +d_2\cos(2n\varepsilon)+\cdots+b_j\sin(jn\varepsilon)+d_j\cos(jn\varepsilon)].$$

When divided by h_0 and truncated at $j=2$, $H(\varepsilon)$ is obtained, i.e.

$$H(\varepsilon)=1+A\sin(n\varepsilon)+b_2\sin(2n\varepsilon)+d_2\cos(2n\varepsilon) \qquad (27)$$

where

$$A=\frac{(h_{\max}-h_0)}{h_0}.$$

Differentiating equation (27) with respect to τ gives

$$\frac{\partial H(\varepsilon)}{\partial\tau}=nA\cos(n\varepsilon)\frac{\partial\varepsilon}{\partial\tau}+2nb_2\cos(2n\varepsilon)\frac{\partial\varepsilon}{\partial\tau} -2nd_2\sin(2n\varepsilon)\frac{\partial\varepsilon}{\partial\tau} \qquad (28)$$

where $\partial\varepsilon/\partial\tau=-\omega Pe$.

Substituting equation (28) into equation (26) gives

$$\frac{\partial Z}{\partial\tau}=\frac{Yn\omega Pe}{H(\varepsilon)^2}[A\cos(n\varepsilon)+2b_2\cos(2n\varepsilon)-2d_2\sin(2n\varepsilon)].$$

The partial derivative $\partial Z/\partial X$ is evaluated by differentiating $Z=Y/H(\varepsilon)$ with respect to X, i.e.

$$\frac{\partial Z}{\partial X}=\frac{-Y\dfrac{\partial H(\varepsilon)}{\partial X}}{H(\varepsilon)^2}.$$

A final expression for the derivative is obtained as

$$\frac{\partial Z}{\partial X}=\frac{-YnPe}{H(\varepsilon)^2}[A\cos(n\varepsilon) +2b_2\cos(2n\varepsilon)-2d_2\sin(2n\varepsilon)].$$

The normal component of the velocity V_y is computed by first writing the continuity equation:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0 \quad\text{or}\quad \frac{-v_0}{Peh_0}\frac{\partial V_x}{\partial X}=\frac{v_0}{Peh_0}\frac{\partial V_y}{\partial Y}$$

(in dimensionless form)

therefore

$$V_y=-\int\frac{\partial V_x}{\partial X}\partial Y$$

using equation (21) and integrating gives

$$V_y=-\sum_{k=1}^{2}\frac{\acute{a}_k(\varepsilon)}{k+1}Y^{k+1}.$$

The coefficient $\acute{a}_k(\varepsilon)$ is obtained by differentiating equations (22) and (23) with respect to ε. Substituting the partial derivatives of Z, and V_x, and V_y in equation (25) enables $dZ/d\tau$ to be evaluated. $H(\varepsilon)$ is calculated from equation (27). The constants of equations (22), (23) and (27) together with the wave parameters n, ω and α as calculated by Penev *et al.* [14] and quoted by

Beschkov and Boyadjiev [15] are used in this study. The boundary conditions remain the same as in the case of smooth flow, while, as mentioned before, the solution to the smooth flow problem becomes the initial distribution of the wavy flow problem.

The finite difference method

An implicit finite difference method was used to approximate equations (19) and (20). This was chosen to be the Alternating Direction Implicit (ADI) method of [19]. This method amounts to taking a half time step $\Delta\tau/2$ using backward difference approximation which is implicit in the X-direction only using the past values in the Z-direction along the grid line $X = X_i$. The next step of the ADI method counters the bias introduced above by using a backwards difference approximation that is implicit in the Z-direction only, using the past values in the X-direction along the grid line $Z = Z_j$ to advance the final half step in time. Thus a full time step is completed.

Putting

$$F_1 = V_x \quad F_2 = \frac{\mathrm{d}Z}{\mathrm{d}\tau} \quad \text{and} \quad F_3 = \frac{1}{H(\varepsilon)^2}$$

equation (19) becomes

$$\frac{\partial\gamma}{\partial\tau} + F_1\frac{\partial\gamma}{\partial X} + F_2\frac{\partial\gamma}{\partial Z} = F_3\frac{\partial^2\gamma}{\partial Z^2} \qquad (29)$$

and applying the ADI method, equation (29) becomes

$$\frac{(\gamma^*_{i,j}-\gamma^k_{i,j})}{\frac{\Delta\tau}{2}} + \frac{F_1}{2\Delta X}(\gamma^*_{i+1,j}-\gamma^*_{i-1,j}) + \frac{F_2}{2\Delta Z}(\gamma^k_{i,j+1}-\gamma^k_{i,j-1})$$

$$= \frac{F_3}{(\Delta Z)^2}(\gamma^k_{i,j-1}-2\gamma^k_{i,j}+\gamma^k_{i,j+1}) \qquad (30)$$

where * denotes the intermediate values calculated at time $\tau = \tau_k + (\Delta\tau/2)$.

Putting

$$F_4 = -\frac{F_1}{2\Delta X} \quad F_5 = \frac{2}{\Delta\tau} \quad F_6 = -F_4$$

$$F_7 = \left(\frac{F_2}{2\Delta Z} + \frac{F_3}{(\Delta Z)^2}\right) \quad F_8 = \left(\frac{2}{\Delta\tau} - \frac{2F_3}{(\Delta Z)^2}\right)$$

$$F_9 = \left(\frac{F_3}{(\Delta Z)^2} - \frac{F_2}{2\Delta Z}\right) \quad F_{10} = \left(\frac{2}{\Delta\tau} + \frac{2F_3}{(\Delta Z)^2}\right)$$

and rearranging equation (30) gives

$$F_4\gamma^*_{i-1,j} + F_5\gamma^*_{i,j} + F_6\gamma^*_{i+1,j} = F_7\gamma^k_{i,j-1} + F_8\gamma^k_{i,j} + F_9\gamma^k_{i,j+1}. \qquad (31)$$

Equation (31) is the first step of the ADI method and is used to calculate all the intermediate values γ^*. The procedure by which this is accomplished is referred to as the "horizontal sweep". In particular for each fixed index, j, $1 \leqslant j \leqslant N-1$, the tri-diagonal system (31) is solved for $\gamma^*_{1,j}, \gamma^*_{2,j}, \ldots, \gamma^*_{M-1,j}$. Thus in order to complete a horizontal sweep an $N-1$ tri-diagonal systems each of the size $M-1$ is solved obtaining $(N-1)(M-1)$ intermediate values.

Upon completion, the second and final step of the ADI method known as the "vertical sweep" could be executed. Equation (29) can be written as:

$$\frac{(\gamma^{k+1}_{i,j}-\gamma^*_{i,j})}{\Delta\tau/2} + \frac{F_1}{2\Delta X}(\gamma^*_{i+1,j}-\gamma^*_{i-1,j})$$

$$+ \frac{F_2}{2\Delta Z}(\gamma^{k+1}_{i,j+1}-\gamma^{k+1}_{i,j-1}) = \frac{F_3}{(\Delta Z)^2}(\gamma^{k+1}_{i,j-1}-2\gamma^{k+1}_{ij}+\gamma^{k+1}_{i,j+1})$$

which is rewritten as

$$-F_7\gamma^{k+1}_{i,j-1} + F_{10}\gamma^{k+1}_{i,j} - F_9\gamma^{k+1}_{i,j+1}$$

$$= -F_4\gamma^*_{i-1,j} + F_5\gamma^*_{i,j} - F_6\gamma^*_{i+1,j}. \qquad (32)$$

Equation (32) is an $M-1$ tri-diagonal system which has $N-1$ equations. So for each index i $(1 \leqslant i \leqslant M-1)$, equation (32) is solved for the final values of $\gamma^{k+1}_{i,1}, \gamma^{k+1}_{i,2}, \ldots, \gamma^{k+1}_{i,N-1}$, at $\tau = \tau + \Delta\tau$. The same procedure is applied to the energy equation whereby a final pair of equations is obtained

$$Ft_4\theta^*_{i-1,j} + Ft_5\theta^*_{i,j} + Ft_6\theta^*_{i+1,j}$$

$$= Ft_7\theta^k_{i,j-1} + Ft_8\theta^k_{i,j} - Ft_9\theta^k_{i,j+1} \qquad (33)$$

$$-Ft_7\theta^{k+1}_{i,j-1} + Ft_{10}\theta^{k+1}_{i,j} - Ft_9\theta^{k+1}_{i,j+1}$$

$$= -Ft_4\theta^*_{i+1,j} + Ft_5\theta^*_{i,j} - Ft_6\theta^*_{i+1,j} \qquad (34)$$

where

$$Ft_2 = \frac{\mathrm{d}Z}{\mathrm{d}\tau} \quad Ft_3 = \frac{F_3}{Le} \quad Ft_4 = F_4$$

$$Ft_5 = F_5 \quad Ft_6 = -F6 \quad Ft_7 = \left(\frac{Ft_2}{2\Delta Z} + \frac{2Ft_3}{(\Delta Z)^2}\right)$$

$$Ft_8 = \left(\frac{2}{\Delta\tau} - \frac{2Ft_3}{(\Delta Z)^2}\right) \quad Ft_9 = \left(\frac{Ft_3}{(\Delta Z)^2} - \frac{Ft_2}{2\Delta Z}\right)$$

$$Ft_{10} = \left(\frac{2}{\Delta\tau} + \frac{2Ft_3}{(\Delta Z)^2}\right).$$

Gauss elimination technique was used to solve the tri-diagonal systems (31)–(34). At a given location, X, both γ and θ oscillate with time at a fixed amplitude and with the same frequency as that of the wave. The amplitude of oscillation diminishes with increasing X as the driving forces $c_{\mathrm{eq}} - c_{\mathrm{i}}$ and $T_{\mathrm{eq}} - T_{\mathrm{i}}$ decrease with the distance. The amplitude of the concentration variation is larger than that of the temperature as the effect of waves on mass transfer is larger than their effect on heat transfer.

RESULTS AND DISCUSSION

Improvement of concentration and temperature

The results present a comparison between the smooth and wavy flows in terms of the non-dimensional concentration (γ) and temperature (θ) averaged

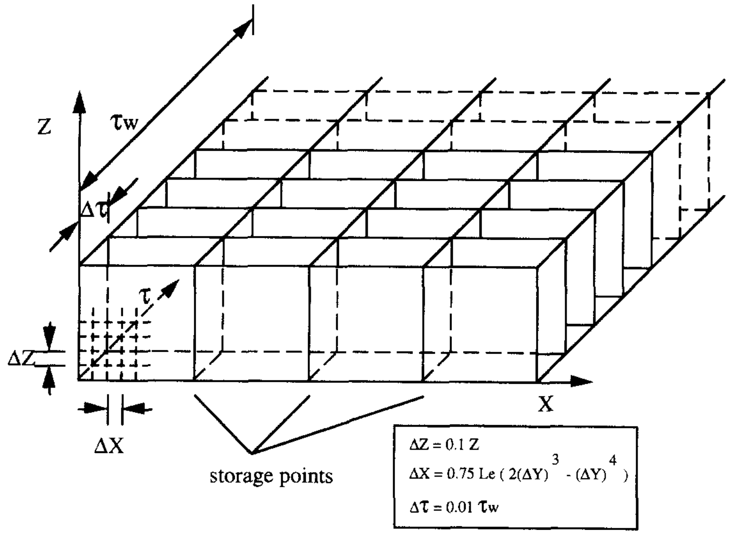

Fig. 2. The segment configuration.

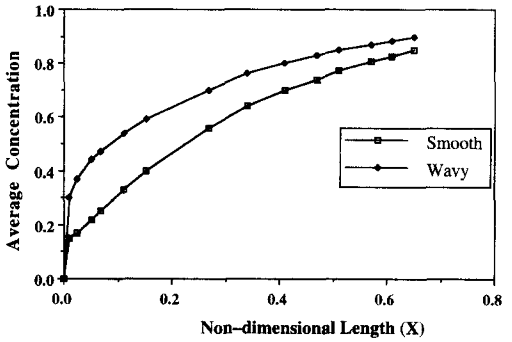

Fig. 3. Comparison of wavy and smooth concentration ($Re = 100, Ha = 0.1, Le = 0.01$).

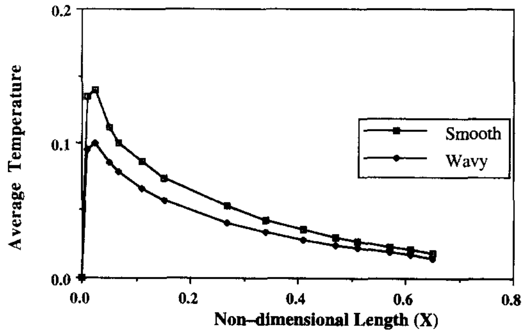

Fig. 4. Comparison of wavy and smooth temperature ($Re = 100, Ha = 0.1, Le = 0.01$).

over the film thickness [$h(\varepsilon)$ in the case of wavy flow and h_0 in the smooth film] and over a complete non-dimensional periodical time (τ_w) which is calculated as follows:

The wavelength (λ) and the wave velocity (v_w) are respectively expressed as

$$\lambda = \frac{2\pi h_0}{n} \quad \text{and} \quad v_w = \alpha v_0$$

the periodical time, $t_w = \lambda / v_w$ and (in dimensionless form)

$$\tau_w = \frac{D t_w}{h_0^2} = \frac{2\pi}{n Pe \alpha}$$

τ_w was divided into a one hundred small $\Delta\tau$'s and the solution was averaged over the whole period. To reduce computer memory space requirement, the solution was not obtained as a continuous field and the X, Z, τ domains were divided into segments (Fig. 2) along the X-axis with only the values at the end of each segment stored and used as inlet conditions to the next one. The Reynold's number and the Schmidt number were taken to be 100 and 500, respectively.

Figures 3 and 4 show the relative improvements of mass and heat transfer of the wavy flow over the smooth one, i.e. a higher comparative concentration and lower comparative temperature. The improvement is greater in concentration than in temperature, which agrees with the published data [4, 20]. This is because the mass is transferred across the gas–liquid interface where the waves have greater effect, while heat is transferred across the solid wall from the solution to the cooling medium where the effect of waves is at minimum. The concentration curves rise gradually towards equilibrium while the temperature curves rise steeply at the beginning due to the rapid absorption and the heat release associated with it, then they start to cool down after reaching a maximum and decay gradually towards the equilibrium.

Effect of the heat of absorption and Lewis number

It is evident from Figs. 5 and 6, respectively that the concentration improves when decreasing the heat of absorption, *Ha*, and increasing the Lewis number, *Le*. Figure 7 shows that the temperature increases with increasing *Ha*, while Fig. 8 shows that it reacts in a mixed way to *Le*; the non-dimensional average temperature increases then decreases with non-dimen-

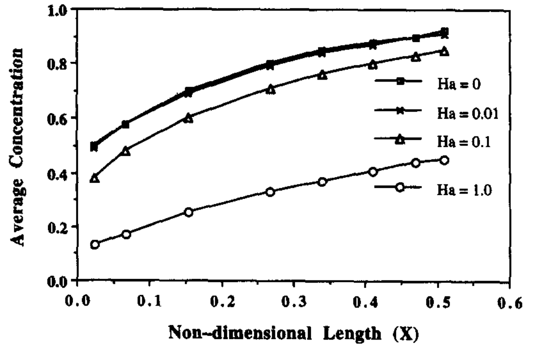

Fig. 5. Variation of concentration with *Ha* for wavy flow ($Re = 100, Le = 0.01$).

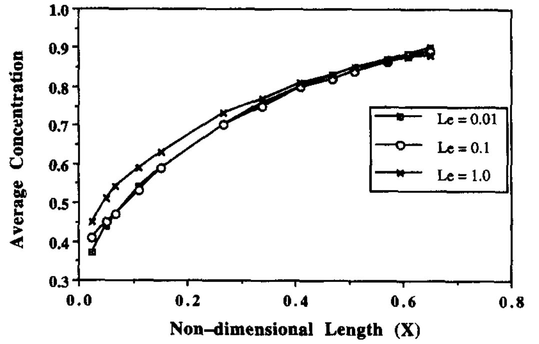

Fig. 6. Variation of concentration with *Le* for wavy flow ($Re = 100, Ha = 0.1$).

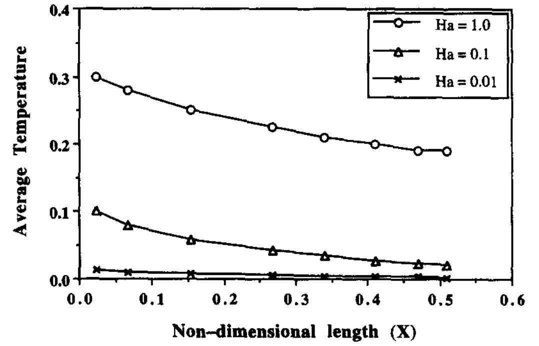

Fig. 7. Variation of temperature with *Ha* for wavy flow ($Re = 100, Le = 0.01$).

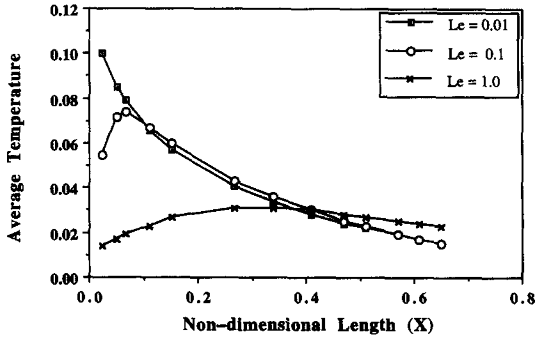

Fig. 8. Variation of temperature with *Le* for wavy flow ($Re = 100, Ha = 0.1$).

sional length. Figure 8 shows that the curve with the lowest *Le* value of 0.01 has the greatest gradient. This is to be expected since liquids with low values of *Le* have higher values of thermal diffusivities (at constant diffusion coefficients) and therefore the change in temperature of such liquids is more pronounced. The temperature reaches different peak values at different *Le* numbers. At low *Le* number the peak temperature occurs closed to the inlet of the absorber. In practice higher concentrations and lower temperatures are desirable which suggests the desirability of lower *Ha* and higher *Le*. It has been pointed out [21] that the refrigeration capacity is more strongly proportional to *Ha* than it is to *Le* and it is dependent on *Le* only by the dependency of *Le* on the concentration variation. Reference [21] concluded that although increasing *Ha* will decrease the concentration variation its net effect will be to increase the refrigeration capacity.

The independent increase and decrease of *Ha* and *Le* is not, in fact, possible as they are both properties of the absorbent at given conditions and are, therefore, interrelated, as indicated by the following expression.

$$Ha = \frac{\Delta h}{C_p}\frac{c_{eq} - c_i}{T_{eq} - T_i} Le.$$

It implies that a higher *Le* will produce a higher *Ha*, which is fortunate from the refrigeration point of view since, in the light of the findings of Ibrahim and Vinnicombe [21] and the findings of this work, a higher *Ha* and a higher *Le* are desirable.

CONCLUSIONS

The analysis of the effect of absorbent's wave motion on the heat and mass transfer supports the hypothesis that at low Reynolds number the improvement of concentration and temperature is not due to the mixing effect of waves, since the velocity profile employed denies such possibility, but rather due to the vertical component of velocity (which is absent in smooth flows) and the convection associated with it. The effects of *Ha* and *Le* on the mass and heat transfer were studied and it was found that a lower *Ha* and a higher *Le* result in a better concentration variation. However, for a higher refrigeration capacity a higher *Ha* is recommended.

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